SPECIAL ARTICLE

PARALLEL SPIRAL CURVES

by W. Zajaczkowski*

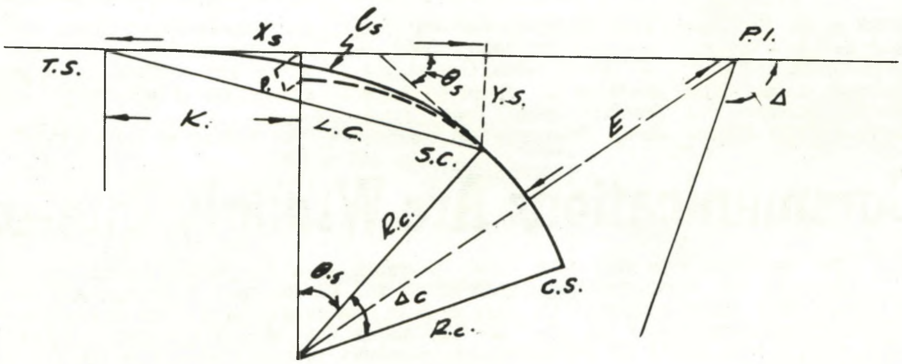
The method of computing parallel** spiral curves is not uniform. Methods vary considerably in different textbooks.

The "Curve Problems" by W. J. Baird, very popular among surveyors, uses the "trial and error" method for computing the length of inner and outer spirals for highway transition curves. A long and tedious way indeed and not very accurate. It is done on assumption that "p" (found in spiral tables) would have the same value for inner and outer spirals as it has for the centre line spiral.

The following formula may be used,

(a) $l_s = 10\sqrt{\frac{24p}{D}}$

to replace the "trial and error" method. It is based also on the assumption that "p" is equal for centre line, inner and outer spirals.



l_s - length of spiral, l_i and l_o - length of inner and outer spirals

R_c - radius of circular curve

D - degree of curve, arc definition, in radians; if the curvature is defined by the chord, D must be computed into arc def.

θ_s - the spiral angle

Y_s - coordinate of point SC

The derivations of formula (a). Using the general formula for "p"

(b) $p = Y_s - R_c \text{ vers. } \theta_s$

and substituting

$$Y_s = \frac{l_s\theta_s}{3}$$

with sufficient accuracy and expanding Cos θ_s to $(1 - \frac{\theta_s^2}{2})$ only, because further terms would become negligible,

$$p = \frac{l_s\theta_s}{3} - R_c + R_c - \frac{R_c\theta_s^2}{2}$$

** Actually two spirals can never be strictly parallel. The definition "Equidistant" would have been better, but is not in use.

simplifying

$$6p = 2(l_s\theta_s) - 3R_c\theta_s^2$$

substituting

$$\theta_s = \frac{l_sD}{200} \text{ and } \theta_s^2 = \frac{l_s^2D^2}{40000}$$

$$6p = \frac{2(l_s^2D)}{200} - \frac{3R_cl_s^2D^2}{40000}$$, but R_c x D (in radians) = 100

$$\therefore \quad 6p = \frac{l_s^2}{100} - \frac{3(l_s^2D)}{400}$$

simplifying

$$2400p = l_s^2\ (4D-3D) = l_s^2D$$

solving for l_s

$$l_s = 10\sqrt{\frac{24p}{D}} \text{ (in radians)}$$

Using the figures in solution 4.02 (see pages 234-236 of "Curve Problems") and comparing results, we find them nearly identical.

R_c inner curve = 768.51, corresponding D = 0.1301219

R_c outer curve = 868.51, corresponding D = 0.1151397

p = 3.18 (but if "p" is used to five decimals, as in tables, p = 3.17895, l_i and l_o will be shorter by 0.04

Using formula (a)	by "trial and error"
$l_i = 10\sqrt{\frac{24\text{x}3.18}{0.1301219}} = 242.18$	$l_i = 242.25$
$l_o = 10\sqrt{\frac{24\text{x}3.18}{0.1151397}} = 257.46$	$l_o = 257.5$

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